32.2 A skylight is comprised of two layers of $\frac{1}{2}$ inch thick laminated glass with a 1 inch air gap $\left(R = 0.5\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}\right)$ in between. The inside and outside film coefficients are $1.5\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}$ and $2.0\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}$, respectively. The outside design temperature is $10^\circ F$ and the inside conditions are $70^\circ F$ with 60% relative humidity. What is the maximum allowable thermal conductivity of the glass to prevent condensate from forming on the inside of the skylight?

A. $0.04\frac{Btu}{hr \cdot ft \cdot {}^\circ F}$

B. $0.08\frac{Btu}{hr \cdot ft \cdot {}^\circ F}$

C. $0.2\frac{Btu}{hr \cdot ft \cdot {}^\circ F}$

D. $0.9\frac{Btu}{hr \cdot ft \cdot {}^\circ F}$

The inside conditions are known. Determine the Dew Point temperature using the Psychrometric Chart. This is the minimum temperature on the inside surface of the glass before which condensate will form.

$$T_{i,db} = 70^\circ F$$

$$\phi_i = 60\%$$

$$T_{i,dp} = 55.6^\circ F$$

Let T_i be the inside temperature.
Let A denote the inner surface of the skylight.
Let B denote the interface between the inner glass and the air gap.
Let C denote the interface between the outer glass and the air gap.
Let D denote the outer surface of the skylight.
Let T_o be the outside temperature.

Review the Reference Handbook section on Conduction. Treat the skylight as a Composite Plane Wall, where the total resistance, R_{total}, can be expressed for this particular situation as:

$$R_{total} = \frac{1}{h_i} + \frac{L}{k} + R + \frac{L}{k} + \frac{1}{h_o}$$

where h_i and h_o are the inside and outside film coefficients, respectively, the $\frac{L}{k}$ terms correspond to conduction through the thickness L of the both sheets of glass having thermal conductivity k, and R represents the resistance of the air gap, which is given. Substitute and simplify to derive an expression for the total resistance, R_{total}, as a function of thermal conductivity, k. Make sure units for each term are in alignment.

$$R_{total} = \frac{1}{1.5\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}} + \frac{\left(\frac{1}{2}in\right)\left(\frac{1ft}{12in}\right)}{k} + 0.5\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu} + \frac{\left(\frac{1}{2}in\right)\left(\frac{1ft}{12in}\right)}{k} + \frac{1}{2.0\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}}$$

$$R_{total} = 1.67\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu} + \frac{1ft}{12\,(k)}$$

The inside temperature will be minimized when the heat loss through the skylight is maximized, i.e. when the heat flux through the "composite wall" is large. Consider the Convection between the inside ambient space at temperature T_i, and the inside of the skylight at surface A. The heat transfer by convection is given by:

$$\dot{Q} = h_i A \Delta T$$

Since the area us unknown, divide by area and work with the heat transfer *per unit area*, or heat flux:

$$\dot{q} = h_i \Delta T$$

$$\dot{q}_{max} = \left(1.5\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}\right)(70^\circ F - 55.6^\circ F) = 21.6\frac{Btu}{hr \cdot ft^2}$$

Considering the entire wall again, find the overall heat transfer coefficient, $U_{overall}$, inclusive of all layers and resistances by assuming the maximum heat flux through the resistance of all layers. The heat flow through the wall can be represented as:

$$\dot{Q} = UA\Delta T$$

Again, we need not be concerned with the area. Divide by area and work with the heat flux:

$$\frac{\dot{Q}}{A} = \dot{q} = U\Delta T$$

Assume the maximum heat flux as previously calculated, and use the inside and outside temperature given to calculate U:

$$U = \frac{\dot{q}_{max}}{\Delta T} = \frac{21.6\frac{Btu}{hr \cdot ft^2}}{70^\circ F - 10^\circ F} = 0.36\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}$$

The Total Resistance, R_{total}, is the inverse of the Overall Coefficient of Heat Transfer, U.

$$U = \frac{1}{R_{total}} \rightarrow R_{total} = \frac{1}{U}$$

$$R_{total} = \frac{1}{0.36\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}} = 2.77\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}$$

Set the calculated value for total resistance equal to the expression for total resistance derived for the composite wall, and solve for the thermal conductivity:

$$R_{total} = 1.67\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu} + \frac{1ft}{12\,(k)} = 2.77\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}$$

$$\frac{1ft}{12\,(k)} = 1.11\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}$$

$$k = \frac{\left(\frac{1ft}{12}\right)}{\left(1.11\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}\right)} = .075\frac{Btu}{hr \cdot ft \cdot {}^\circ F}$$

Answer B